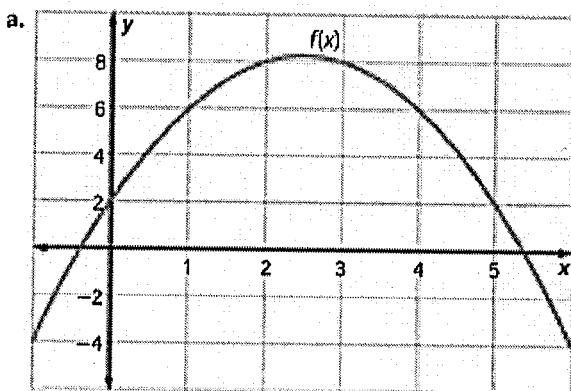


## Polynomials Unit 5 Lesson 1

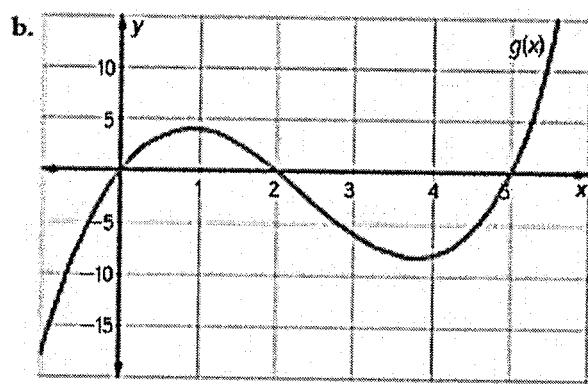
- 1** In Parts a-d, use a curve-fitting tool and/or algebraic reasoning to find rules for functions  $f(x)$ ,  $g(x)$ ,  $h(x)$ , and  $j(x)$  that model the given graph patterns. In each case, report the graph points used as the basis of your curve-fitting, the rule of the modeling function, and your reasons for choosing a model of that type.



Points  $(0, 2), (5, 2), (1, 6), (4, 6)$

Quad Reg

$$f(x) = -x^2 + 5x + 2$$



Points  $(0, 0), (2, 0), (5, 0), (1, 4)$

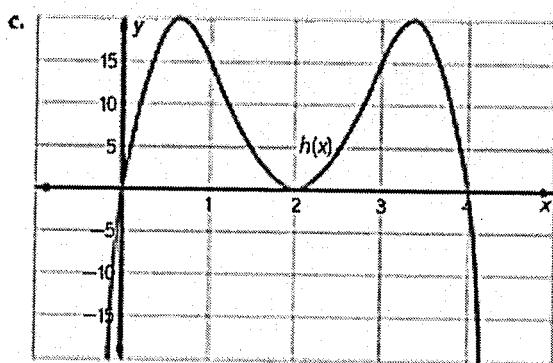
$$y = a(x-2)(x-5)$$

$$4 = a(1)(1-2)(1-5)$$

$$4 = a(4)$$

$$\frac{4}{4} = \frac{4}{4}$$

$$a = 1 \quad g(x) = x(x-2)(x-5)$$



$(0, 0), (2, \text{Max}), (4, 0), (1, 15)$

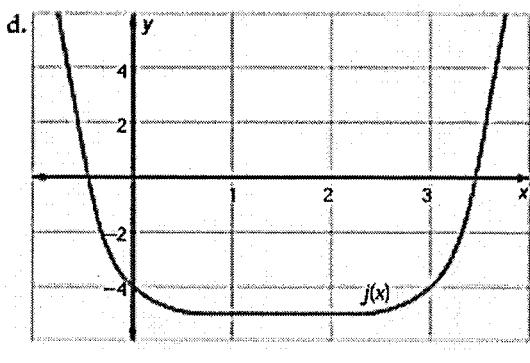
$$y = a(x-4)(x-2)^2$$

$$15 = a(1-4)(1-2)^2$$

$$\frac{15}{-3} = \frac{-3a}{-3}$$

$$a = -5$$

$$h(x) = -5x(x-4)(x-2)^2$$



$$(0, -4)(1, -5)(2, -5)(3, -4)$$

$$(3.5, 0)$$

Too Flat for Quadratic so  
I did Quart Reg.

$$j(x) = .476x^4 - 2.857x^3 + 5.738x^2 - 4.357x - 4$$



Graph each function and then calculate or estimate coordinates of all:

- local maximum points.
- local minimum points.
- $x$ -intercepts.
- $y$ -intercepts.

a.  $f(x) = 2x^2 + 4x + 1$

$y\text{-int } (0, 1)$

$x = \frac{-4}{2(2)} = -1$

$2(-1)^2 + 4(-1) + 1 = -1$

$x = \frac{-4 \pm \sqrt{16 - 4(2)(1)}}{4}$

$x = -1 \pm \frac{\sqrt{8}}{2} = -1 \pm \frac{\sqrt{2}}{2}$

$(-1 + \frac{\sqrt{2}}{2}, 0)$   $x\text{-int}$   
 $(-1 - \frac{\sqrt{2}}{2}, 0)$

b.  $g(x) = x^3 + 2x^2 + 3x + 7$

$y\text{-int } (0, 7)$

No local Max or Mins

$x$ -intercepts

$(-2.132494, 0)$

Calc  
zero  
on graph

c.  $h(x) = x^3 - 6x^2 + 12x - 8 = (x-2)^3$

$y\text{-int } (0, -8)$

No local Max or Mins

$x$ -intercept  $(2, 0)$  Mult 3

From Table

d.  $s(x) = x^4 - 8x^3 + 20x^2 - 16x$

$y\text{-intercept } (0, 0)$

$x$ -intercepts  $(0, 0), (2, 0), (4, 0)$   
from  
graph & Table

local Max  $(2, 0)$  From graph.

local Min  $(3.4142105, -4)$  Calc  
 $(3.4142105, -4)$  Min

For each algebraic expression:

- write an equivalent expression in standard polynomial form.
- identify the degrees of the expressions being combined and the degree of the result.

a.  $(2x^2 + 5x - 2) + (-2x^2 + 3x + 7) = 8x + 5$   
Degree 2      Degree 2      Degree 1

b.  $(2x^2 + 5x - 2) - (5x + 7) = 2x^2 - 9$   
Degree 2      Degree 1      Degree 2

c.  $(-7x^3 + 6x^2 + 3x - 7) - (3x^4 + 7x^3 + 4x^2 - 3x + 2) = -3x^4 - 14x^3 + 2x^2 + 6x - 9$   
Degree 3      Degree 4      Degree 4

d.  $(2x^2 + 3x - 7 + 5x^5 - 3x^4 - 7x^3) + (3x^5 + 2x^4 + 2x^3 + 4x^2 + 6x + 1) =$   
Degree 5      Degree 5

$$8x^5 - x^4 - 5x^3 + 6x^2 + 9x - 6$$

Degree 5

e.  $(5x^5 - 3x^4 + 2x^2 + 6x - 7) + (7x^3 - 2x^2 - 3x + 5) = 5x^5 - 3x^4 + 7x^3 + 3x - 2$   
Degree 5      Degree 3      Degree 5

Without using a graph, table, or CAS, find all zeroes of these functions. Then describe the end behaviour of each function.

a.  $f(x) = (x - 3)(x + 4)$

$$(x-3)(x+4)=0$$

$$x-3=0 \quad x+4=0$$

$$x=3 \quad x=-4$$

End behavior

as  $x \rightarrow \pm\infty \quad y \rightarrow \infty$

positive  
leading  
coefficient

b.  $g(x) = (x^2 - 9)x$

$$x(x^2-9)=0$$

$$x(x+3)(x-3)=0$$

$$x=0 \quad x+3=0 \quad x-3=0$$

$$x=-3 \quad x=3$$

End behavior

as  $x \rightarrow -\infty \quad y \rightarrow -\infty$

as  $x \rightarrow +\infty \quad y \rightarrow +\infty$

positive  
leading  
coefficient

c.  $h(x) = (x^2 + 5)x^2$

$$x^2=0 \quad x^2+5=0$$

$$x=0 \quad x^2=-5$$

$$\text{Multi} \Rightarrow \quad x = \pm 5i$$

End behavior

as  $x \rightarrow \pm\infty \quad y \rightarrow \infty$

positive leading  
coefficient

d.  $s(t) = -(t + 5)^2$

$$0 = -\frac{(t+5)^2}{1}$$

$$\sqrt{0 = (t+5)^2}$$

$$t+5=0 \quad t = -5_{\text{mult}} 2$$

End behavior

as  $x \rightarrow \pm\infty \quad y \rightarrow -\infty$

Negative  
leading  
coefficient

For each algebraic expression:

- write an equivalent expression in standard polynomial form.
- identify the degrees of the expressions being combined and the degree of the result.

a.  $(7x + 3)(x - 1)$

Degree 1      Degree 1

$$7x^2 - 7x + 3x - 3$$

$$7x^2 - 4x - 3$$

Degree 2

b.  $(3x + 5)^2 x^2$  (Degree 1)<sup>2</sup> • Degree 2

$$(3x+5)(3x+5) = 9x^2 + 30x + 25$$

$$x^2(9x^2 + 30x + 25) = 9x^4 + 30x^3 + 25x^2$$

Degree 4

c.  $(2x^2 + 3x - 7)(3x + 7)$  (Degree 2)(Degree 1)

$$\begin{array}{r} 3x(2x^2 + 3x - 7) = 6x^3 + 9x^2 - 21x \\ + 7(2x^2 + 3x - 7) \quad + 14x^2 + 21x - 49 \\ \hline 6x^3 + 23x^2 - 49 \\ \text{Degree 3} \end{array}$$

d.  $(7x^3 - 6x + 4)(2x^2 - 7)$  (Degree 3)(Degree 2)

$$\begin{array}{r} 2x^2(7x^3 - 6x + 4) = 14x^5 - 12x^3 + 8x^2 \\ + -7(7x^3 - 6x + 4) \quad + -49x^3 \quad + 42x - 28 \\ \hline 14x^5 - 61x^3 + 8x^2 + 42x - 28 \quad \text{Degree 5} \end{array}$$

e.  $(7x^3 + 2)(5x^2 + 3x - 8)$  (Degree 3)(Degree 2)

$$\begin{array}{r} 7x^3(5x^2 + 3x - 8) = 35x^5 + 21x^4 - 56x^3 + 10x^2 + 40x - 16 \\ 2(5x^2 + 3x - 8) \quad \text{Degree 5} \end{array}$$

f.  $(-3x^4 + 2x^2 + 6x)(7x^3 - 2x^2 + 5)$  (Degree 4)(Degree 3)

$$\begin{array}{r} 7x^3(-3x^4 + 2x^2 + 6x) = -21x^7 + 14x^5 + 42x^4 \\ -2x^2(-3x^4 + 2x^2 + 6x) = +6x^6 \quad -4x^4 - 12x^3 \\ 5(-3x^4 + 2x^2 + 6x) = + -15x^4 \quad + 10x^2 + 30x \\ \hline -21x^7 + 6x^6 + 14x^5 + 23x^4 - 12x^3 + 10x^2 + 30x \\ \text{Degree 7} \end{array}$$

7. The *Galaxy Sport and Outdoor Gear* company has a climbing wall in the middle of its store. Before the store opened for business, the owners did some market research and concluded that the daily number of climbing wall customers would be related to the price per climb  $x$  by the linear function  $n(x) = 100 - 4x$ .

a. According to this function, how many daily climbing wall customers will there be if the price per climb is \$10? What if the price per climb is \$15? What if the climb is offered to customers at no cost?

$$\begin{array}{ll} n(10) = 100 - 4(10) & n(15) = 100 - 4(15) \\ = 60 & = 40 \end{array}$$

At No Cost  
there would  
be 100  
Customers.

60 customers for \$10  
per climb      40 customers for \$15  
per climb

b. What do the numbers 100 and  $-4$  in the rule for  $n(x)$  tell about the relationship between climb price and number of customers?

at No Cost per climb there would be 100 customers  
and for each dollar more charged per climb  
you would have 4 less customers.

c. What is a reasonable domain for  $n(x)$  in this situation? That is, what values of  $x$  are plausible inputs for the function?

$$0 \leq x \leq 25$$

$\downarrow$   
I wouldn't pay more than \$25  
to climb the wall.

d. What is the range of  $n(x)$  for the domain you specified in Part c? That is, what are the possible values of  $n(x)$  corresponding to plausible inputs for the function?

$$0 \leq n(x) \leq 100$$

e. If the function  $I(x)$  tells how daily income from the climbing wall depends on price per climb, why is  $I(x) = 100x - 4x^2$  a suitable rule for that function?

$$\begin{aligned} \text{Income} &= (\text{price per climb})(\text{Number of climbs}) \\ &= x(100 - 4x) \\ &= 100x - 4x^2 \end{aligned}$$

So this Makes Sense

-  The function  $e(x) = 2x + 150$  shows how daily operating expenses for the Galaxy Sport climbing wall (Task 7) depend on the price per climb  $x$ .

- a. Write two algebraic rules for the function  $P(x)$  that gives daily profit from the climbing wall as a function of price per climb, (1) one that shows how income and operating expense functions are used in the calculation of profit and (2) another that is in simpler equivalent form.

$$P(x) = (100x - 4x^2) - (2x + 150) \quad ①$$

$$= -4x^2 + 98x - 150 \quad ②$$

- b. Find  $P(5)$ . Explain what this result tells about climbing wall profit prospects.

$$P(5) = -4(5)^2 + 98(5) - 150$$

$$= 240$$

If you charge \$5 per climb you will make a daily profit of \$240.

- c. What is a reasonable domain for  $P(x)$  in this problem situation?

$$0 \leq x \leq 25$$

- d. What is the range of  $P(x)$  for the domain you specified in Part c?

$$-200 \leq P(x) \leq 450.25$$

- f. Find the price(s) that will yield maximum daily profit from the climbing wall.

Max profit of \$450.25 occurs when price per climb is \$12.25

